



Graphs of mod 1000 on powers of 2 to 9

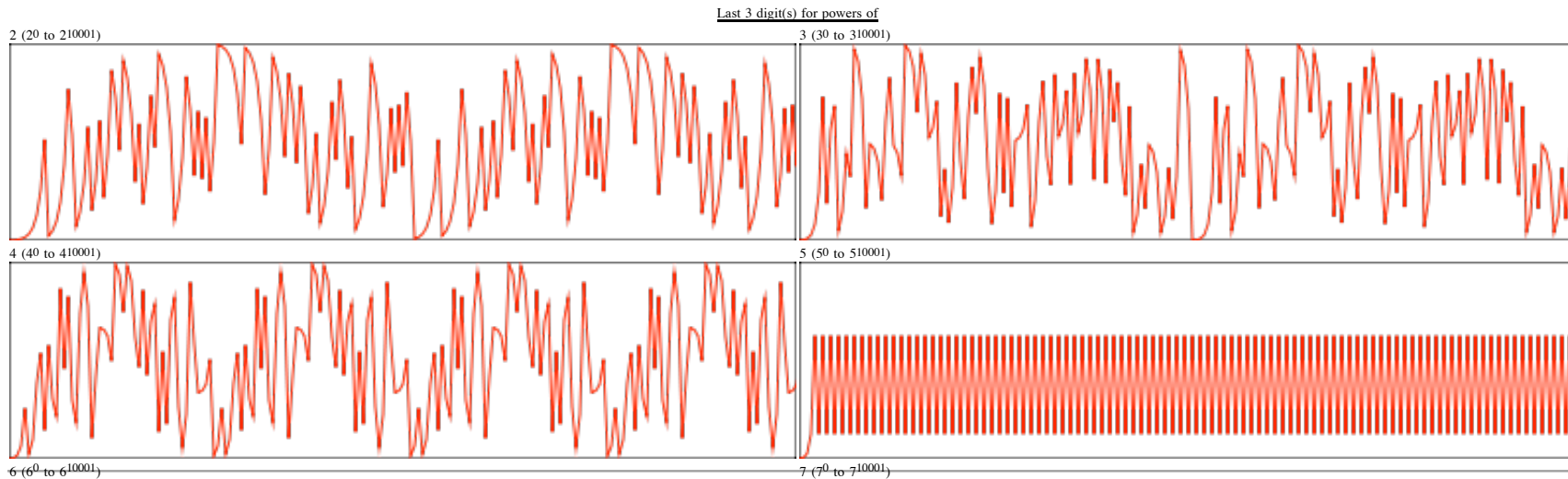
- Exponents = {0,1,2,... 10001 }
- Objects = { 2, 3, 4, 5, 6, 7, 8, 9, }
- Object_Power_Sets = { {x^{exp}} | x ∈ Objects ; exp ∈ Exponents }
- Cycle_Sets = { {o mod 1000} | o ∈ O ; O ∈ Object_Power_Sets }

Elements of Cycle_Sets

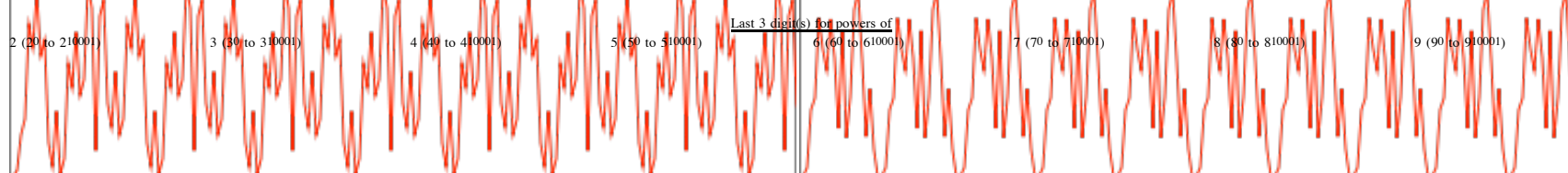
These sets form a Group under the operation $\otimes_{1000}(x,y) = (x * y) \text{ mod } 1000$. For identity element derivation [see below](#).

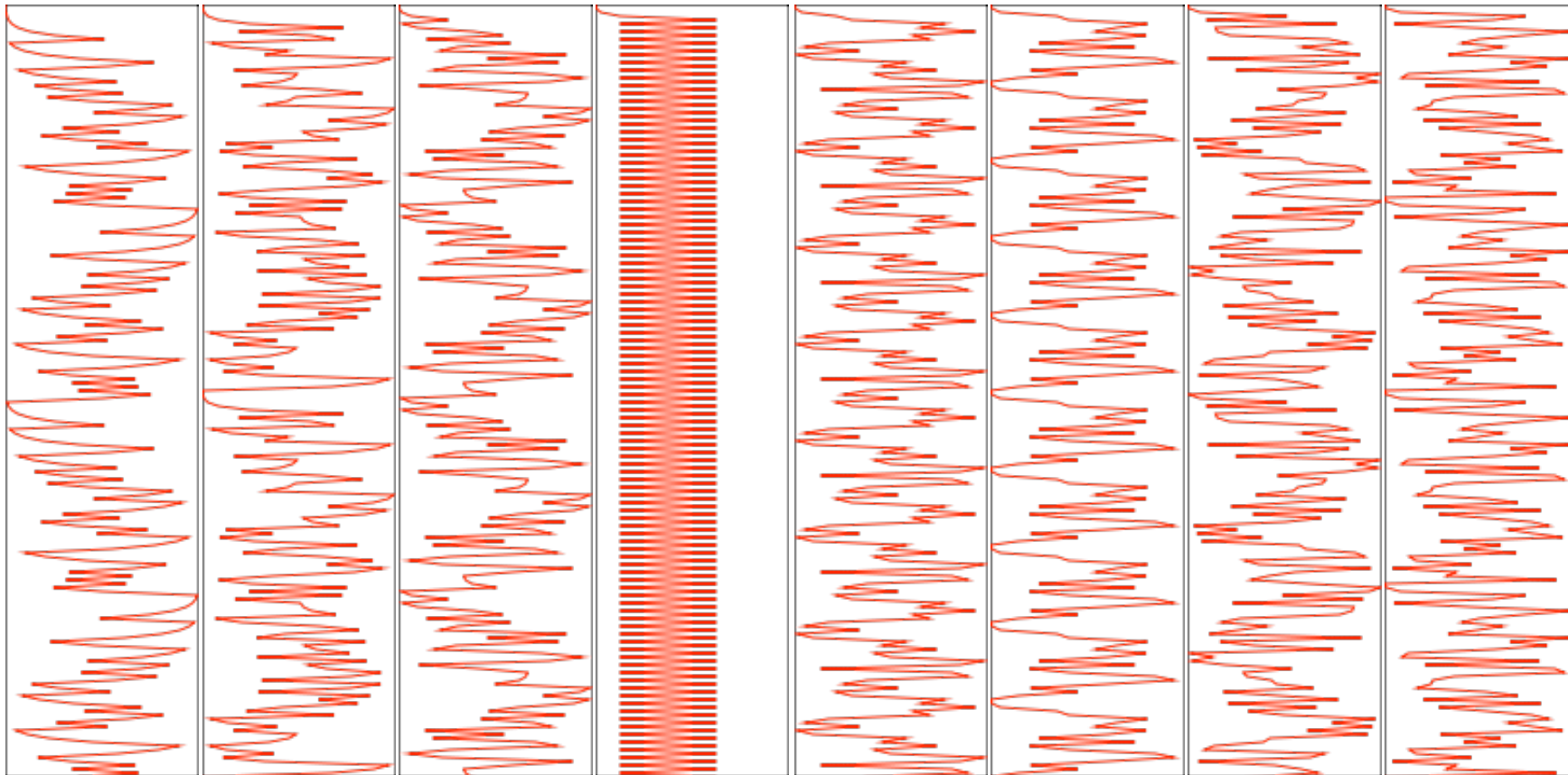
- Cycle_Set₂ = {8,16,32,64,128,256,512,24,48,96,192,384,768,536,72,144,288,576,152,304,608,216,432,864,728,456,912,824,648,296,592,184,368,736,472,944,888,776,552,104,208,416,832,664,328,656,312,624,248,496,992,984,968,936,872,744,488,976,952,904,808,616,232,464,928,856} [count=100] ([show table](#))
- Cycle_Set₃ = {1,3,9,27,81,243,729,187,561,683,49,147,441,323,969,907,721,163,489,467,401,203,609,827,481,443,329,987,961,883,649,947,841,523,569,707,121,363,89,267,801,403,209,627,881,643,929,787,361,83,249,747,241,723,169,507,521,563,689,67,201,603,809,427,281,843,521} [count=100] ([show table](#))
- Cycle_Set₄ = {16,64,256,24,96,384,536,144,576,304,216,864,456,824,296,184,736,944,776,104,416,664,656,624,496,984,936,744,976,904,616,464,856,424,696,784,136,544,176,704,816,264,56,224,896,584,336,344,376,504} [count=50] ([show table](#))
- Cycle_Set₅ = {125,625} [count=2] ([show table](#))
- Cycle_Set₆ = {216,296,776,656,936,616,696,176,56,336,16,96,576,456,736,416,496,976,856,136,816,896,376,256,536} [count=25] ([show table](#))
- Cycle_Set₇ = {1,7,49,343,401,807,649,543,801,607,249,743,201,407,849,943,601,207,449,143} [count=20] ([show table](#))
- Cycle_Set₈ = {8,64,512,96,768,144,152,216,728,824,592,736,888,104,832,656,248,984,872,976,808,464,712,696,568,544,352,816,528,224,792,336,688,504,32,256,48,384,72,576,608,864,912,296,368,944,552,416,328,624,992,936,488,904,232,856,848,784,272,176,408,264,112,896,168,3} [count=100] ([show table](#))
- Cycle_Set₉ = {1,9,81,729,561,49,441,969,721,489,401,609,481,329,961,649,841,569,121,89,801,209,881,929,361,249,241,169,521,689,201,809,281,529,761,849,641,769,921,289,601,409,681,129,161,449,41,369,321,889} [count=50] ([show table](#))

Horizontal View (back to top)



Vertical View (back to top)





Identity element

The members in the cycle set (say x, y, z) will be a subset of Numbers = $\{0, 1, 2, 3, \dots, 98, 99, \dots, 1000\}$.
 With the operator $\otimes_{1000} (x, y) = (x * y) \bmod 1000$, the search for identity element proceeds as follows:

- Let x be the identity element of a cycle set, then for any y from the cycle set, $(x * y) \bmod 1000 = y$. This is possible with $x=1+u$ ($u \in \text{Numbers}$). Then

$$\begin{aligned} (x*y) \bmod 1000 &= (y + u*y) \bmod 1000 \\ &= y \bmod 1000 + (u*y) \bmod 1000 \\ &= y + (u*y) \bmod 1000 \end{aligned}$$
 need to find smallest u such that $u*y$ to be a multiple of 1000, then $(u*y) \bmod 1000 = 0$
- An element y from Cycle_Set₂ or Cycle_Set₄ or Cycle_Set₆ or Cycle_Set₈ is a multiple of 8,
 then $(x*y) \bmod 1000$

$$\begin{aligned} &= (y + 8^u * u * z) \bmod 1000 \\ &= y \bmod 1000 + (8^u * u * y) \bmod 1000 \\ &= y + (8^u * u * y) \bmod 1000 \end{aligned}$$
 need $8^u * u * y$ to be multiple of 1000, irrespective of y (member of cycle set)
 that is possible with $u=125$ ($x=126$) or $u=250$ ($x=251$) or $u=375$ ($x=376$), of this $x=376$ ($u=375$) is a member of the cycle sets
 $\therefore 376$ is the identity element for the cycle sets mentioned above
- For Cycle_Set₃ or Cycle_Set₇ or Cycle_Set₉,
 then $(x*y) \bmod 1000$

$$\begin{aligned} &= (y + u*y) \bmod 1000 \\ &= y \bmod 1000 + (u*y) \bmod 1000 \\ &= y + (u*y) \bmod 1000 \end{aligned}$$
 need $u*y$ to be multiple of 1000, irrespective of y (member of cycle set)
 that is possible with $u=0$ ($x=1$) or $u=999$ ($x=1000$), but $x=1000$ ($u=999$) is not a member of these sets

$\therefore I$ is the identity element for the cycle sets mentioned above

- Elements of Cycle_Sets are multiples of 125.

then $(x^y) \bmod 1000$

$$= (y + 125^k u^z) \bmod 1000$$

$$= y \bmod 1000 + (125^k u^z) \bmod 1000$$

$$= y + (125^k u^z) \bmod 1000$$

need u^z to be multiple of 1000, irrespective of y (member of cycle set)

that is possible with $u=624$ ($x=625$) a member of the cycle set

$\therefore 625$ is the identity element for the Cycle_Sets